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Profile of Students' Visual Thinking in Understanding and Applying Derivative Concepts

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A R T I C L E I N F O

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A B S T R A C T

The aim of this research is to obtain students' visual thinking profiles in understanding and applying derivative concepts. Qualitative research methods are used with the steps determining the subject, developing questions, collecting data, interpreting, presenting, reducing, categorizing, coding, and triangulating data, analysing data, and drawing conclusions. The subjects of this research were students of the mathematics education study program of PGRI Madiun University. At first, students understand the concept of derivatives without using visual thinking skills. Students understand the concept of derivative as a demotion. After learning, students understand the concept of derivatives with images as visualization. Students apply the concept of guidance without using visual thinking. After learning, students can apply derivative concepts by using images as visualization. Students understand that they must apply derivative concepts and explain with pictures or graphs. Students plan by analysing and then using function graphs to explain it. Students carry out plans by calculating analytically and completing explanations using pictures or graphs. Students check again by paying attention to the answers from friends beside them and discussions with the lecturer.

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INTRODUCTION

Visual thinking is the activity of the mind processing visual information. Wahyuni, G., Destini, R., & Mujib, A. (2023) explained that visual thinking is the ability to think to process and formulate information obtained as data so that it can be used to represent it in the form of images or graphics. Darmadi, D., & Handoyo, B. (2016) defines visual thinking as thinking using visual codes, namely codes or information that can be presented in the form of images/graphs. Olivia, F. (2013) defines that visual thinking as a way of thinking used by many talented people in history. Ali, W. (2018) defines visual thinking as an intuitive intellectual process and visual imagination ideas.

Visual information in the mind is often called mental imagery. Knowledge related to visual thinking is important because it can illustrate the accuracy or strength of understanding. Some mathematical concepts will be easier to understand by using visualization. Darmadi, D. (2015) has examined the visual thinking profile of male prospective mathematics teacher students in understanding the formal definition of a convergent sequence. Darmadi (2019) has researched the model of visual thinking of prospective mathematics teachers to understand the formal definition of convergent sequences based on gender differences.

Some math problems will be easier to solve if you use visualization. Visual thinking is a skill that can be useful for students. Yogi, A., & Nurdin, N. (2021) have examined the visual thinking abilities of prospective teacher students in solving geometric problems. Darmadi, D., & Handoyo, B. (2016) have examined the visual thinking profile of prospective mathematics teacher students with a visual learning style in solving trigonometry problems.

Students are the nation's next generation. Students need visual thinking skills to be able to understand and solve problems better. By having visualizations or images, understanding a concept or definition will be much better than not using visualizations or images. By having a visualization or picture, the application of a concept or definition will be much more stable in its explanation. Surya, E. (2012) has researched that visual thinking in maximizing students' mathematics learning can build national character. However, Darmadi, D. (2017) has identified students' visual thinking errors in drawing real function graphs.

To face the challenges of the times, students are required to have the skills and knowledge to understand. Ability or understanding skills to learn, discuss with other people, see market potential, and so on. Ginanjar, A. Y. (2019) explains the importance of mastering mathematical concepts in solving mathematical problems in elementary school. The research results of Dewi, S. Z., & Ibrahim, T. (2019) explain the importance of understanding concepts to overcome misconceptions in science learning materials in elementary schools. Diva, D. F., Andriyani, J., Rangkuti, S. A., Prasiska, M., Tobing, T. E. W. L., Irani, A. R., & Saragih, R. M. B. (2023) explains the importance of understanding geogebraic concepts in mathematics learning.

Apart from the ability to understand, the ability to apply concepts is also needed in everyday life. These abilities or skills are basic skills or abilities that are often used after understanding. Concepts are abstract ideas. The concept stated can be in the form of a definition. Fujiawati, F. S. (2016) defines the concept as an image that shows the structure of an object. Gunawan, G., Harjono, A., & Sutrio, S. (2015) defines a concept as an abstraction that describes the general characteristics of an object. Kania, N. (2018) defines concepts as abstract ideas.

One of the concepts studied in mathematics lessons is the concept of derivatives. Outside of mathematics, derivatives have many meanings. The descent when climbing a mountain or on a mountain slope shows a downward slope. Derivation during the exam shows students' dishonesty during the exam. Heredity can also be interpreted as carrying genes in the field of animal husbandry or biology. The concept of derivatives needs to be understood and also needs to be applied. Maulana, F. I. (2021) explains the application of derivatives in determining maximum profit in the furniture industry using Maple. Asyhar, B. (2014) explains the application of derivatives in maximum profit analysis problems.

There has been no research that describes students' visual thinking in understanding and applying derivative concepts. Therefore, the aim and problem formulation of this research is to determine the visual thinking profile of students in understanding and applying derivative concepts. An overview of students' visual way of thinking in understanding and applying an important concept to obtain a learning basis that makes it easier for students to understand and apply concepts. Apart from that, students can also learn to make it easier to understand and apply a concept to solve problems.

METHODOLOGY

To obtain students' thinking profiles in understanding and applying derived concepts, qualitative research was carried out. The methods for this research are determining the subject, developing questions, collecting data, interpreting, presenting, reducing, categorizing, coding, and triangulating data, analysing data, and drawing conclusions.

The subjects of this research were students of the mathematics education study program of PGRI Madiun University. The total number of subjects who participated in the research was 9 students. Subjects are 5th-semester students taking real analysis courses. Incidentally, all the students are female.

The problem chosen is a derivative problem at absolute prices. The task of finding the derivative of an absolute price function is an option because students are not used to it. Previously, students already knew the absolute price function. So, this question only requires students to remember, develop, and explain the concept of derivatives again. Apart from that, students are also asked to apply the concept of derivatives of absolute price functions in the form of images so that they can obtain a visual thinking profile. The task or question that students must do is to remember and explain the concept of derivatives and explain them in the form of pictures. After that, students are asked to apply the concept of derivatives and apply it to determine $f'(0)$ for $f(x) = |x|$ and explain it in the form of pictures or graphs.

Data collection was carried out in two stages. The first stage is to obtain students' visual thinking profiles in understanding derivative concepts. The second stage is more focused on getting students' visual thinking profiles in applying derivative concepts to the questions given. Data was collected using assignment methods and in-depth interviews.

Interpretation is carried out as soon as possible after the researcher obtains the data. The researcher immediately asked again if there was an unclear answer. This is done to confirm that you really get saturated data.

Data presentation is carried out without reducing the naturalness of the data. In presenting the data, data reduction, categorization, coding and triangulation were also carried out. Data reduction is carried out by ignoring data that is not relevant to the research objectives. Categorization is carried out to obtain students' visual thinking stages in understanding and applying derivative concepts. Coding is done to make it easier to trace the data. Time and source triangulation was carried out to obtain valid data.

Data analysis was carried out on valid data. Based on the data from the interpretation and categorization, conclusions can be obtained as a result of the research. Conclusions are drawn more focused on depicting students' visual way of thinking in understanding and applying derivative concepts.

RESEARCH RESULT AND DISCUSSION

Students' visual thinking profile in understanding derivative concepts

To find out students' visual thinking profiles, questions were asked related to derivative concepts. The concept of derivatives according to students is as follows.

- *The derivative is a derivative function contained in the limit function*
- *A function that can be reduced the power with certain variables*
- *The derivative is a measurement of how a function changes as a certain value changes*
- *Variable whose power is* $n-1$
- *Variable whose power is* $n-1$*, from* x^n *to* x^{n-1}
- *A variable whose power is reduced by one, so it can be written* x^{n-1}
- *The derivative shows changes in the variable function with the symbol* $f'(x)$
- *A mathematical function that has variables with powers* $n-1$

Almost all students remember the concept of derivatives from notation and its reduced power by 1. Nearly all (56%) students explain it by stating the power explicitly. However, the other answers also point to the same thing. Further interview results show that students only remember that the derivative of the function $f(x) = x^n$ is $f'(x) = nx^{n-1}$.

Only one student (11%) answered that the derivative is related to the limit function. However, the results of deeper interviews with these students showed that the students only remembered function limits and nothing else. This student's answer is interesting because the derivative of a function is related to the limit.

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ .... (1)

or

$f'(c) = \lim_{x \to c} \frac{f(x)-f(c)}{x-c}$ .... (2)

Students do not have an idea regarding the concept of derivatives. This was obtained based on the results of in-depth interviews with students. Students are asked to provide explanations using pictures. This is done to explore students' visualizations regarding derivatives. However, all students answered that they did not have an idea regarding the concept of derivatives. So after being confirmed again, the student explained that he did not have a picture or visualization regarding derivatives.

The visualization of the derivative concept can be understood from the understanding that the derivative at a point shows the slope of the function at that point. The slope of a function at a point is nothing but the gradient of the tangent line of a function at that point. The gradient of a line is generally symbolized by m where m=Δy/Δx. Since $\Delta y = y_2 - y_1$ and $\Delta x = x_2 - x_1$, then $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Because $y = f(x)$ then the gradient through points x and c is $m = \frac{f(x)-f(c)}{x-c}$. To get the slope at one point, namely point c, you can get it by bringing x closer to c. By the concept of limits (1), the slope at point c is the derivative of the function f at point c. The derivative $f'(c)$ is the slope of the function f at point c. The slope of the function f at point c is nothing but the gradient of the tangent line to the function f at point c. An illustration or visualization of this derivative concept is as explained in Figure 1.

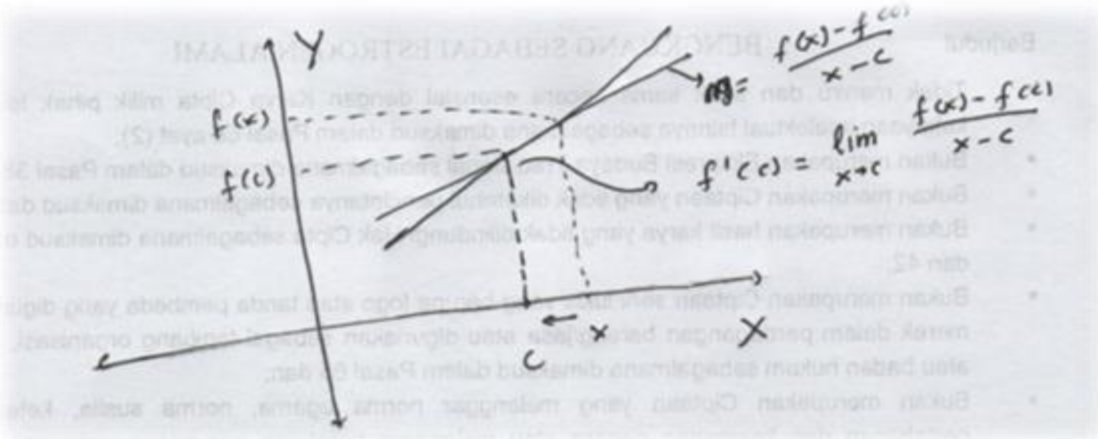

Figure 1. Visualization of the concept of derivative function f at point c

To get the slope of a function at one point, namely point x, you can get it by forming two points, namely $(x, f(x))$ and $(x+h, f(x+h))$. The idea used is to use the h approach to get closer to 0. With the limit concept, the slope of the function f at point x (2). The derivative of the function f at point x or $f'(x)$ is the slope of the function f at point x. The slope of the function f at point x is nothing but the gradient of the tangent line to the function f at point x. An illustration or visualization of this derivative concept is as explained in Figure 2.

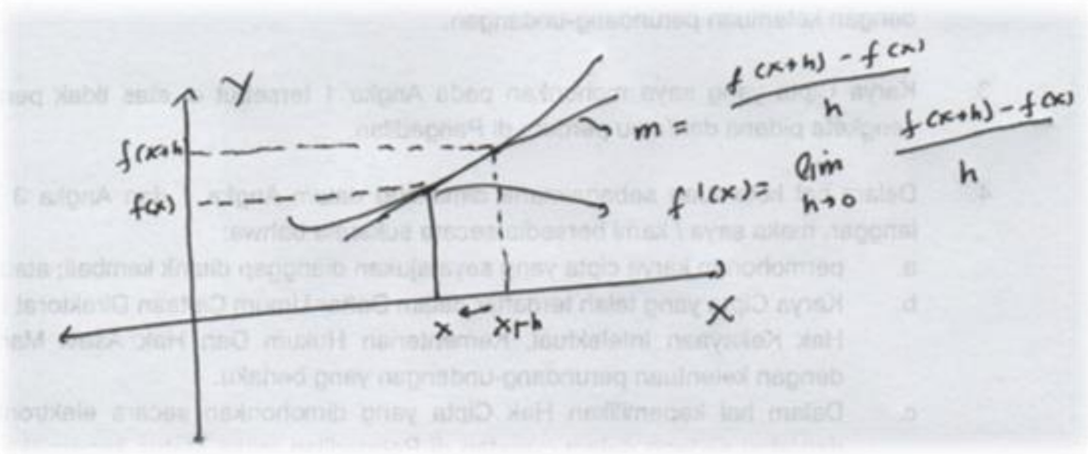

Figure 2. Visualization of the concept of derivative of the function f at point x

Students' visual thinking profile in applying derivative concepts

To obtain students' visual thinking profiles in applying derivative concepts, students were asked questions. The first question is to determine $f'(0)$ for $f(x) = |x|$ and explain using pictures.

Students should already know or have an idea or graphical visualization of the absolute price function.

$$|x| = \begin{cases} x, jika\ x \geq 0 \\ -x, jika\ x < 0 \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

This can be confirmed because students have previously learned by using pictures or graphs of absolute price functions in learning continuous functions. Apart from the concept of continuity, the discussion used images to clarify student visualization.

After getting the concept of the derivative that $f'(c) = \lim_{x \to c} \frac{f(x)-f(c)}{x-c}$, students use this knowledge to solve. Most (60%) students gave the answer $f'(0) = \lim_{x \to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x \to 0} \frac{|x|-0}{x-0} = \lim_{x \to 0} \frac{|x|}{x} = \lim_{x \to 0} \frac{x}{x} = 1$ so $f'(0) = 1$. Some (40%) students gave the following answer $f'(0) = \lim_{x \to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x \to 0} \frac{|x|-0}{x-0} = \lim_{x \to 0} \frac{|x|}{x} = \lim_{x \to 0} \frac{-x}{x} = -1$ so $f'(0) = -1$. There was doubt expressed on the students' faces after the assignments were collected and the answers collected were read out.

Errors in answers given by students can occur due to a lack of accuracy. Students forget that $|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$. Most (60%) students only remember that $|x| = x$. Some (40%) students only remember that $|x| = -x$. Students are less careful that $|x| = x$ if $x \geq 0$ and $|x| = -x$ if $x < 0$. Due to a lack of thoroughness, the answers given by students are less precise.

Errors in answers given by students can occur due to a lack of a strong concept of limits. A function limit is said to exist if the right limit is the same as the left limit. So the correct answer is as follows.

$$f'(0) = \lim_{x\to 0}\frac{f(x)-f(0)}{x-0} = \lim_{x\to 0}\frac{|x|-0}{x-0} = \lim_{x\to 0}\frac{|x|}{x} = \begin{cases}\lim_{x\to 0^+}\frac{x}{x} = 1\\ \lim_{x\to 0^-}\frac{-x}{x} = -1\end{cases}$$

…………………..…… (4)

Because the right limit is not the same as the left limit, the limit value does not exist. Because the limit value does not exist, $f'(0)$ does not exist.

Regardless of whether it is true or not, students are asked to explain using pictures. This is done to get an idea or visualization of the application of the derivative concept to the absolute price function at point 0. Students need a lot of time to answer this question. After confirmation through deeper interviews, it turned out that students did not have an idea or visualization regarding the application of this derivative concept.

Visualization using images shows the level of understanding and application of concepts. The graphic description of the absolute price function has been accepted in the previous discussion. Students just need to explain the application of the concept of derivative $f'(0)$ for $f(x) = |x|$. Visualization or depiction of the derivative $f'(0)$ for $f(x) = |x|$ can be seen in figure 3.

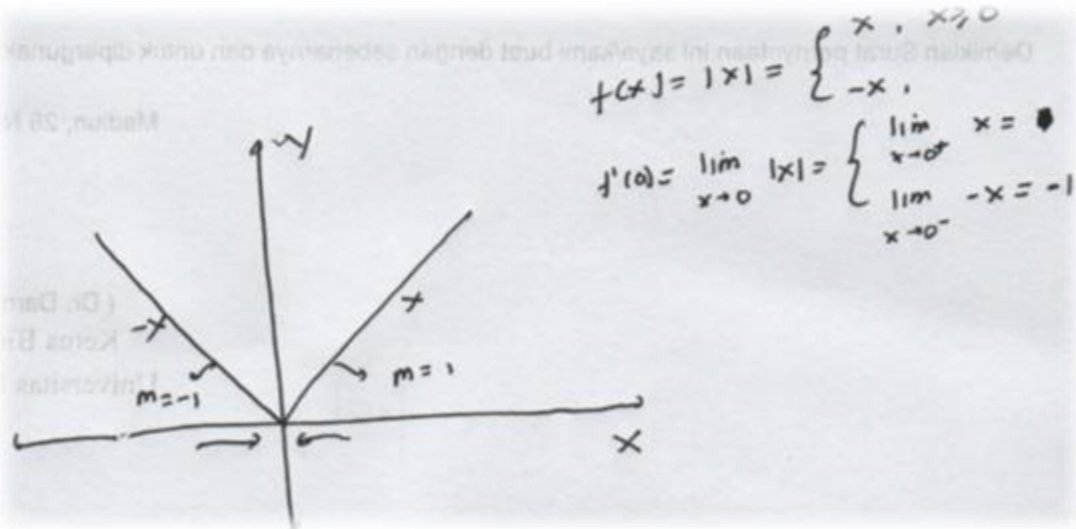

Figure 3. Visualization of the application of the derivative concept $f(x) = |x|$ at point 0.

CONCLUSIONS AND RECOMMENDATIONS

At first, students understand the concept of derivatives without using visual thinking skills. Students understand the concept of the derivative as a demotion. After learning, students better understand the concept of derivatives with images as visualization.

Students apply the concept of guidance without using visual thinking. After learning, students are better able to apply derivative concepts by using images as visualization. Students understand that they must apply derivative concepts and explain with pictures or graphs. Students plan by analysing and then using function graphs to explain it. Students carry out plans by calculating analytically and completing explanations using pictures or graphs. Students check again by paying attention to the answers from friends beside them and discussions with the lecturer.

Students should be diligent in improving their visual thinking abilities and skills in understanding and applying mathematical concepts. Lecturers should often explain using pictures as visualization so that learning can be more meaningful.

ADVANCED RESEARCH

This qualitative research is natural in nature which shows the lack of students' visual thinking abilities and skills in understanding and applying a concept. To improve this, further research is needed regarding the development of learning models that can improve students' visual thinking abilities.

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