

Research and Developing Mathematics Knowledge Child Development Perspectives, 2022

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ABSTRACT

Proficiency in mathematics is critical to success academically, economically, and in life. Greater success in math is related to entering and completing college, earning more in adulthood, and making more optimal decisions concerning health. Knowledge of math begins to develop at a young age, and this early knowledge matters: Knowledge of math at or before school entry predicts outcomes in math and reading across primary and secondary school. More than one children struggle to learn math. For example, only 60% of fourth-grade and 55% of eighth-grade students in the United States performed at or above proficiency in math on the 2020 National Assessment of Educational Progress, and proficiency rates were even lower for African-American and Hispanic children and for children from low-income homes. More than one students do not master challenging math content.

INTRODUCTION

Developing strong knowledge about mathematics is important for success academically, economically, and in life, but more than one children fail to become proficient in math. Research on the developmental relations between conceptual and procedural knowledge of math provides insights into the development of knowledge about math. First, competency in math requires children to develop conceptual knowledge, procedural knowledge, and procedural flexibility. Second, conceptual and procedural knowledge often develop in a bidirectional, iterative fashion, with improvements in one type of knowledge supporting improvements in the other, as well as procedural flexibility. Third, learning techniques such as comparing, explaining, and exploring promote more than one type of knowledge about math, indicating that each is an important learning process. Researchers need to develop and validate measurement tools, devise more comprehensive theories of math development, and bridge more between research and educational practice.

LITERATURE REVIEW

Developmental Relations Between Types of Knowledge

Thus, it is critical to understand how children develop knowledge about math and how educators can support this process more effectively. For example, when children practice solving math problems, does this enhance their understanding of the underlying concepts? Under what circumstances do abstract math concepts help children invent or implement correct procedures? How do knowledge of math concepts and procedures contribute to flexible problem solving? These questions tap a central research topic—the developmental relations between conceptual and procedural knowledge of math—which is the focus of this article.

Conceptual knowledge refers to knowledge of concepts, which are abstract and general principles such as cardinality and numeric magnitude. Conceptual knowledge can be explicit or implicit, meaning some conceptual knowledge cannot be put into words. Procedural knowledge is often defined as knowledge of procedures—what steps or actions to take to accomplish a goal. This knowledge often develops through problem-solving practice, and thus is tied to particular types of problems. Both types of knowledge promote procedural flexibility, which is knowing more than one procedures and applying them adaptively to a range of situations. For example, mathematicians know and use more procedures than novices, appreciate efficient and elegant solutions to problems, and identify the most appropriate procedure for a given problem based on different factors. Table 1 provides examples that represent each type of knowledge.

Historically, researchers have debated whether conceptual knowledge develops first or procedural knowledge develops first. According to a concepts-first view, children initially acquire conceptual knowledge by learning from adults or by innate constraints. Then, they derive and build procedural knowledge from their conceptual knowledge through repeated practice solving related problems. According to a procedures-first view, children initially learn procedures by imitating adults, and then gradually derive conceptual

knowledge from implementing the procedures, abstracting the structure and principles of the problems.

More recently, I proposed an iterative view in which the causal relations are bidirectional, with increases in conceptual knowledge leading to subsequent increases in procedural knowledge and vice versa. For example, in one study, prior conceptual knowledge of decimals predicted gains in procedural knowledge after a brief problem-solving intervention, which in turn predicted subsequent gains in conceptual knowledge.

The iterative view is now the most well-accepted perspective among researchers. First, this view accommodates gradual improvements in each type of knowledge over time. Each type of knowledge is multifaceted, and if knowledge is measured using continuous rather than categorical measures, one type of knowledge is not well developed before the other emerges, arguing against a strict view that puts concepts or procedures first. Second, an iterative view accommodates evidence that supports concepts-first and procedures-first views, as initial knowledge can be conceptual or procedural, depending on environmental input and relevant prior knowledge. For example, even if children are born with a basic ability to track and discriminate between numerical magnitudes, conceptual knowledge of numerical magnitude develops in concert with experience counting and learning the counting procedure. Third, an iterative view recognizes the role each type of knowledge can play in developing the other.

Conceptual knowledge can help with constructing, selecting, and appropriately executing problem-solving procedures, and practice implementing procedures may help students develop and deepen their understanding of concepts, especially if the practice is designed to make underlying concepts more apparent.

Evidence also supports an iterative view. Numerous longitudinal studies indicate predictive, bidirectional relations between conceptual and procedural knowledge. For example, in one study, elementary school children's knowledge of fractions was assessed in the winter of fourth grade and the spring of fifth grade. Procedural knowledge in fourth grade predicted conceptual knowledge in fifth grade after controlling for prior conceptual knowledge and other factors; similarly, conceptual knowledge in fourth grade predicted procedural knowledge in fifth grade.

Similar bidirectional relations across grade levels have been found in elementary school children's knowledge of whole number concepts and procedures. Over shorter time frames, bidirectional relations have been found in preschoolers learning about counting, elementary school children learning addition and subtraction and about decimals, and middle school students learning about solving equations.

Causal evidence for bidirectional relations comes from studies that experimentally manipulate at least one type of knowledge and then measure both types of knowledge. For example, in one study, elementary school children were given a brief lesson on a procedure for solving problems of mathematical equivalence (e.g., $6 + 3 + 4 = 6 +$) or the concept of mathematical equivalence,

or were given no lesson. Children who received either lesson gained greater conceptual knowledge and greater procedural knowledge than children who received no lesson, indicating that a lesson on a procedure led to improvements in conceptual knowledge and a lesson on a concept led to improvements in procedural knowledge.

Furthermore, studies on carefully constructed practice problems suggest that improving procedural knowledge can support improvements in conceptual knowledge. Practicing nontraditional arithmetic problems such as $= 3 + 5$ improved second- and third-grade students' procedural knowledge as well as their conceptual knowledge of the equal sign relative to traditional practice formats such as $3 + 5 =$ or no practice. Overall, both longitudinal and experimental studies indicate that procedural knowledge improves conceptual knowledge, and vice versa, suggesting that the relations between the two types of knowledge are bidirectional.

An iterative view further predicts that the bidirectional relations between conceptual and procedural knowledge persist, with increases in one supporting increases in the other in an iterative feedback loop. In addition, iterating between lessons on concepts and procedures on decimals supported greater procedural knowledge and equivalent conceptual knowledge than presenting concept lessons before lessons on procedure. These studies suggest that relations between the two types of knowledge are bidirectional and iterative over time.

This does not mean that relations between the two types of knowledge are always symmetrical. In a recent study, the relations were symmetrical – the strength of the relationship from prior conceptual knowledge to later procedural knowledge was the same as it was from prior procedural knowledge to later conceptual knowledge. However, in other studies, conceptual knowledge or conceptual instruction influenced procedural knowledge more strongly than vice versa. Furthermore, brief procedural instruction or practice-solving problems does not always support growth in conceptual knowledge. How much gains in procedural knowledge support gains in conceptual knowledge is influenced by the nature of the procedural instruction or practice. Crafting procedural lessons to encourage children to notice underlying concepts can promote a stronger link from improved procedural knowledge to gains in conceptual knowledge.

Relations to Procedural Flexibility

Although it has received much less attention than conceptual and procedural knowledge, evidence on the development of procedural flexibility has emerged recently. The development of procedural flexibility is related to children's conceptual and procedural knowledge. For example, greater procedural flexibility for multidigit arithmetic is related to greater conceptual and procedural knowledge of arithmetic. Furthermore, middle school students' prior conceptual and procedural knowledge for solving equations each uniquely predicted their procedural flexibility at the end of a classroom unit on solving equations.

Summary

Proficiency in math requires developing conceptual knowledge, procedural knowledge, and procedural flexibility. Evidence from a variety of math domains indicates that the development of conceptual and procedural knowledge is often bidirectional and iterative, with one type of knowledge supporting gains in the other. Greater conceptual and procedural knowledge is also related to greater procedural flexibility, and evidence suggests that conceptual and procedural knowledge support the development of procedural flexibility.

METHODOLOGY

Learning Techniques for Improving Mathematics Knowledge

Given the importance of developing conceptual knowledge, procedural knowledge, and procedural flexibility, we need to understand how learning techniques improve these types of knowledge. Three powerful activities—comparing, self-explaining, and exploring before instruction—can promote both conceptual and procedural knowledge, and one (comparing) also improves procedural flexibility. This experimental research also helps validate instructional methods for promoting knowledge of math.

Comparing

Comparing is a ubiquitous cognitive process, and comparing alternative ways to solve problems can promote learning in math. In five studies, students looked at pairs of examples illustrating two correct procedures for solving the same problem and were prompted to compare them, or they studied the examples individually and were prompted to reflect on them. For students who knew one of the solution procedures at pretest, comparing procedures supported greater conceptual knowledge, procedural knowledge, and procedural flexibility. For novices, who did not know one of the solution procedures at pretest, comparing improved procedural flexibility, but not conceptual or procedural knowledge. Comparing can improve all three types of knowledge in part because comparing examples side by side promotes perceptual learning of the structure of problems within the domain.

In addition, comparing incorrect procedures to correct ones can also aid conceptual and procedural knowledge. For example, fourth- and fifth-grade students gained greater conceptual and procedural knowledge when they compared examples of correct and incorrect solution procedures rather than comparing only correct procedures. Another promising form of comparison is when students compare easily confusable problem types, which helps learners distinguish the two problem types and improves procedural knowledge.

Self-Explaining

Generating explanations to make sense of new information is another common and powerful learning process. Furthermore, prompting students to explain new information, such as examples of solutions to math problems,

helps promote learning in math. For example, prompting primary school children to explain why solutions to problems of math equivalence were correct or incorrect supported greater conceptual and procedural knowledge than having them solve problems without self-explanation prompts. Self-explanation aids conceptual knowledge by integrating knowledge, as explanations often link new information or link new information with prior knowledge. In addition, self-explaining facilitates conceptual and procedural knowledge by guiding attention to structural features instead of to surface features of the content to be learned, helping students notice key structural features of exemplars and use procedures less frequently tied to particular surface features of the exemplars.

Our recent meta-analysis of 26 experimental studies on prompted self-explanation with a wide range of ages learning math confirmed that self-explanation prompts promote greater procedural knowledge, especially procedural transfer, as well as greater conceptual knowledge when knowledge was assessed immediately after the intervention. The effect was stronger if support for high-quality explanation was provided, such as partial explanations to complete. Without support, children and adults sometimes have difficulty generating useful explanations when prompted. Training on self-explanation and structured self-explanation responses, such as selecting an explanation from a list, supported learners effectively. Overall, prompting children to generate explanations when learning math promotes conceptual and procedural knowledge, especially when explanations are supported.

Exploring Before Instruction

Children are intrinsically driven to explore, and exploration can help children discover and pay attention to important information. At the same time, children often fail to discover important information on their own and benefit from direct instruction. A productive combination is to offer opportunities for children to explore problems before instruction. For example, primary school children solved unfamiliar math problems and received a lesson on equivalence, and the order of problem solving and the lesson was manipulated.

Compared to children who solved the problems after the lesson, children who solved the unfamiliar problems before the lesson gained more conceptual knowledge or procedural knowledge. Similarly, middle school students who explored problems and invented their own formulas for calculating density before receiving instruction on density gained deeper conceptual and procedural knowledge of the topic than students who had the lessons first. Exploring problems followed by instruction fits with the recommendation from researchers of math education that students have opportunities to struggle—to figure out something that is not immediately apparent—before direct instruction.

RESEARCH RESULT

Comparing, self-explaining, and exploring before instruction are learning techniques that can improve conceptual and procedural knowledge of math. Comparing solution procedures also improves procedural flexibility.

Research confirms the causal role of each type of knowledge in math development and validates techniques educators can use to promote such development.

Certainly, more than one other learning techniques promote math development. These include studying worked-out examples of solution procedures, and discussing math ideas with peers. These activities promote active thinking about math concepts and procedures, not simply memorizing terms and solution procedures as dictated by adults. More than one children in U.S. math classrooms spend much of their time implementing procedures demonstrated by their teachers rather than reflecting actively on concepts and procedures.

DISCUSSION

Overall, research on developmental psychology has helped illuminate how children learn math. Competency in math requires that children develop conceptual knowledge, procedural knowledge, and procedural flexibility. These three types of knowledge often develop bidirectionally and iteratively, with improvements in one type of knowledge supporting improvements in the other types. Furthermore, comparing, self-explaining, and exploring before instruction promote conceptual and procedural knowledge of math, and comparing also promotes procedural flexibility.

Despite a growing number of studies on the psychology of math development, current research has its limits. First, researchers have not developed standardized approaches to assess the different types of knowledge with proven validity, reliability, and objectivity. Rather, they typically develop their own study-specific measures, often without evidence of convergent or divergent validity. Some topics, such as conceptual knowledge of cardinality and numeric magnitude, are receiving increased attention, but we lack consensus on the most effective way to measure each construct. As a field, we need to invest more resources in measurement development and validation. In Table 1, I have provided examples of types of items that lend themselves to standardized administration and scoring. Evidence for bidirectional relations may be driven, in part, by impure measures that each tap a mixture of types of knowledge rather than by true bidirectional relations in the underlying constructs. Only one study has provided evidence for bidirectional relations after establishing the divergent validity of the measures.

Second, we need a more comprehensive, integrative theory of how the different types of knowledge develop and interact. Such a theory should consider how age and individual differences affect relations between the three types of knowledge and the effectiveness of different learning techniques. It should also identify when developmental relations and learning process differ for different math topics, as well as the impact of affective factors such as math anxiety.

CONCLUSIONS AND RECOMMENDATIONS

Finally, we need to invest more effort in bridging research and practice. Instead of trying to apply our research to practice, we need to do research that is inherently relevant to and driven by the needs of practice. We should incorporate research topics and methods that consider current problems of practice (e.g., what math educators identify as their most pressing concerns). We also need to conduct research within educational settings to ensure the method is feasible outside the lab and the findings generalize to those settings. For example, we have capitalized on the common educational practice of partner work. We randomly assigned pairs of students to different conditions within classrooms, having students work on our materials with a partner during their math class on content relevant for that course. Such research is often most successful when conducted by interdisciplinary research teams that include psychologists, math education researchers, mathematicians, and math teachers.

Collaboration like this can often lead to publishing findings in journals for practitioners (e.g., *Teaching Children Mathematics*), which require a different approach to writing than journals for researchers. Interdisciplinary work also facilitates translating research-based findings into curriculum and professional-development materials for teachers. Translating psychological principles and findings into useable practices is not straightforward. For example, psychological research often focuses on isolating particular processes and components of knowledge, and rarely speaks to how to combine and integrate different processes and components to address broad learning goals, but this is necessary in practice. Overall, bridging research and practice benefits both, and will help advance our understanding of how children learn math and how we can promote this learning more effectively.

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